

# Lefschetz zeta function and some flows associated with circle valued Morse maps on 3-manifolds

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Jiang and Wang introduced the twisted Lefschetz zeta function  $\zeta_{f,\rho}$  for a map  $f$  in [3]. Let  $S$  be a surface with a boundary and  $T_f$  the mapping torus obtained from a homeomorphism  $f : S \rightarrow S$ . They observed that  $\zeta_{f,\rho}$  coincides with the twisted Alexander polynomial of  $T_f$ ,  $A_{T_f,\rho}$ , which was defined by Lin [4].

In this talk, we discuss a generalization of this observation to the following setting. Let  $M$  be an orientable 3-manifold with the Betti number  $b_1(M) > 0$  and  $f$  a Morse map  $M \rightarrow S^1$ . We define the twisted Novikov homology and obtain the torsion  $\tau_{M,f,\rho}$  of the homology. We show:

$$\tau_{M,f,\rho}(t) \cdot \zeta_{f,\rho}(t) = A_{M,\rho}(t).$$

Note that if  $M = T_f$ , the torsion  $\tau_{M,f,\rho}(t)$  is equal to 1.

This result may be regarded as the twisted version of an equation of Hutchings-Lee [2]. Mark [5] showed that the Seiberg- Witten equation leads to the Hutchings-Lee equation for a symmetric flow obtained from a circle valued Morse map on  $M$ . We define a Heegaard splitting corresponding to the Morse map, and then give concrete examples of computations of  $\tau_{M,f,1}$  and  $\zeta_{M,f,1}$  using sutured manifold theory [1]. This is the first non-trivial example as far as I know.

## REFERENCES

1. D. Gabai, *Detecting fibred links in  $S^3$* , Comment. Math. Helv. 61 (1986), 519–555.
2. M. Hutchings and Y-J. Lee, *Circle-valued Morse theory, Reidemeister torsion, and Seiberg-Witten invariants of 3-manifolds*, Topology, 38 (1999), 861-888.
3. B. Jiang and S. Wang, *Twisted topological invariants associated with representations*, Topics in knot theory (Erzurum, 1992), 211–227, NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., 399, Kluwer Acad. Publ., Dordrecht, 1993.
4. X. Lin, *Representations of knot groups and twisted Alexander polynomials*, Acta Math. Sin., Engl. Ser. 17 (2001), 361-380.
5. T. Mark, *Torsion, TQFT, and Seiberg-Witten invariants of 3-manifolds*, Geom. Topol. 6 (2002), 27-58.

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