Algebraic varieties via the Kauffman bracket skein module

FUMIKAZU NAGASATO

In this talk, for a knot K in 3-sphere S^3 we define an algebraic variety $\mathcal{F}(K)$ in a complex space $\mathbb{C}^N$ in the following steps. For a braid presentation σ of a knot K, we first construct finitely many polynomials $\{p_{\sigma,i}\}_i$ on $\mathbb{C}^N$ by using an action of σ to the Kauffman bracket skein module (KBSM) of a handlebody at $t = -1$. Then we consider the ideal $\mathcal{SL}(\sigma)$ generated by the polynomials $\{p_{\sigma,i}\}$. Since the common zeros of $\mathcal{SL}(\sigma)$ are the solutions of the algebraic equations $\{p_{\sigma,i} = 0\}$, they give a variety in $\mathbb{C}^N$. The variety turns out to be invariant under the Markov moves and thus becomes a knot invariant. This is the algebraic variety $\mathcal{F}(K)$.

By using Bullock's theorem (quantization of the $SL(2,\mathbb{C})$-character variety), the variety $\mathcal{F}(K)$ can be considered as a variety containing "a section" of the $SL(2,\mathbb{C})$-character variety of the knot group. This view point gives relationships of the variety $\mathcal{F}(K)$ with the maximal degree of the A-polynomial $A_K(m,l)$ in terms of l, the number of $SL(2,\mathbb{C})$-irreducible metabelian characters of the knot group and the Casson-Lin invariant (the knot signature).

On the other hand, the quotient ring $\mathcal{K}(\sigma) := \mathbb{C}[x_1,\cdots,x_N]/\mathcal{SL}(\sigma)$ has a natural filtration

$$\mathbb{C} = \mathcal{K}^{(1)}(\sigma) \subset \mathcal{K}^{(2)}(\sigma) \subset \mathcal{K}^{(3)}(\sigma) = \mathcal{K}(\sigma).$$

The filters are also invariant under the Markov moves and thus become knot invariants. (Hence we denote $\mathcal{K}^{(i)}(\sigma)$ by $\mathcal{K}^{(i)}(K)$.) The first filter is trivial invariant. Then it turns out that the second filter corresponds to the degree 0 knot contact homology.

Tokyo Institute of Technology