

## The kinkiness of knots and smooth four-manifolds

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In [2], R.Gompf introduced the notion of the kinkiness of a knot. The positive (resp. negative) kinkiness  $\kappa_+(\kappa_-)$  of a knot  $K$  is the minimum, over all normally-immersed disks in a 4-ball  $B^4$  whose boundary is  $K$ , of the number of positive (negative) self-intersections of such a disk. The positive (resp. negative) unknotting number  $u_+(u_-)$  of a knot  $K$  is the minimum, over all sequences transforming  $K$  to the unknot, of the number of positive (negative) crossings which are changed.

Fact 1.  $u \geq u_+ + u_- \geq \kappa_+ + \kappa_-$ ,  $u_+ \geq \kappa_+$ ,  $u_- \geq \kappa_-$ .

Fact 2.  $u \geq c^* \geq \kappa_+ + \kappa_- \geq |\sigma|/2$ ,  $c^* \geq g^* \geq |\sigma|/2$ .

Here  $u$ ,  $c^*$ ,  $g^*$  and  $\sigma$  are the unknotting number, the 4-dimensional clasp number, the slice genus and the signature of a knot. Let  $K$  be the pretzel knot  $\varphi(-3, 5, 7)$ . We know that  $u_+(K) = \kappa_+(K) = \sigma(K) = 0$  and  $\kappa_-(K) = c^*(K) = g^*(K) = 1$ .

*Problem.* Is it true that  $u_-(\varphi(-3, 5, 7)) = 2$ ?

In particular, the pretzel knot  $K$  bounds null-homologous disk which is properly smoothly embedded in  $\overline{CP^2} - \text{Int}B^4$ . On the other hand, we show that  $K$  does not bound such a disk in  $\#_{i=1}^m CP^2 - \text{Int}B^4$  for any positive integer  $m$  via gauge theory [1]. We also give some application to 4-dimensional topology. We show the existence of exotic smooth structures on all noncompact, smooth 4-submanifolds of  $CP^2$ .

### References

- [1] S. K. Donaldson, An application of gauge theory to four-dimensional topology, J. Differential Geom. 18 (1983), no. 2, 279-315.
- [2] R. Gompf, Smooth concordance of topologically slice knots, Topology 25 (1986), no. 3, 353-373.

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