Talks: Titles and Abstracts

The 9th Autumn Workshop on Number Theory
Hakuba, Japan

November 6–10, 2006

1 Survey Talks

Kaoru Hiraga (Kyoto University):
Endoscopy on GSp(4)

Abstract: This will be a survey of endoscopy. Following the theme of this workshop, the case of GSp(4) will be illustrated mainly. Conjectures on endoscopy, such as fundamental lemmas, transfer conjectures, A-packets, and the multiplicity formula will be explained.

References :

1. J. Arthur, *Unipotent automorphic representations: conjectures*, in *Orbites unipotentes et représentations* II, Astérisque 171-172 (1989), 13–71.

2. J. Arthur, *Unipotent automorphic representations: Global Motivation*, in *Automorphic forms, Shimura varieties, and L-functions*, Vol. I (Ann Arbor, MI, 1988), 161–209, Perspect. Math., 10, Academic Press, Boston, MA, 1990.

Both are available from "Arthur archive":
http://www.claymath.org/cw/arthur/index.php

Atsushi Ichino (Osaka City University):

On integral representations of automorphic L-functions for GSp(4)

Abstract: Among other means, an integral representation gives the meromorphic continuation of an automorphic L-function. In this talk, recalling the doubling method as an example, I will try to give some feeling for the techniques. It is often the case that unique models play a role. In the case of GSp(4), I will review the basic fact about the Whittaker model and the Bessel model.

Tetsushi Ito (Kyoto University):

On motives and ℓ-adic Galois representations associated to automorphic representations of GSp(4)

Abstract: The so-called Langlands conjecture predicts a relation between automorphic representations of $\mathrm{GSp}(4)/\mathbb{Q}$, motives, and 4-dimensional ℓ-adic representations of the absolute Galois group of $\mathbb{Q}$. First, I will briefly recall the case of $\mathrm{GL}(2)/\mathbb{Q}$ (i.e. the case of modular curves and elliptic modular forms) as a motivating example. I will explain how to calculate the Hasse-Weil zeta function of Siegel 3-folds by comparing the Arthur-Selberg and the Lefschetz trace formulae. Then, I will explain such a calculation enables us to construct motives and ℓ-adic Galois representations associated to automorphic representations of GSp(4). If time permits, I will also indicate several important applications (Taylor-Wiles systems, generalized Ramanujan-Petersson conjecture, etc.). Hopefully, this talk will be an introduction to basic concepts, and further applications will be given in other talks of this workshop.

References :

1. *Automorphic Forms, Representations, and L-Functions 1 & 2* (A. Borel and W. Casselman, ed), Proc. Symp. Pure Math., 55, Amer. Math. Soc., 1979

   Available from *AMS Books Online*:

   http://www.ams.org/online_bks/pspum331/

   http://www.ams.org/online_bks/pspum332/

2. *Formes automorphes. II. Le cas du groupe* GSp(4) (Tilouine, J., Carayol, H., Harris, M., Vigneras, M.-F., ed), Asterisque No. 302 (2005).

3. Blasius, D., Rogawski, J., *Zeta functions of Shimura varieties*, in *Motives* (Seattle, WA, 1991), 525–571, Proc. Symp. Pure Math., 55, Amer. Math. Soc., 1994.

4. Kottwitz, R., *Shimura varieties and λ-adic representations*, in *Automorphic forms, Shimura varieties, and L-functions*, Vol. I (Ann Arbor, MI, 1988), 161–209, Perspect. Math., 10, Academic Press, Boston, MA, 1990.

Kazuko Konno (Fukuoka University of Education) :
Induced representations of GSp(4) *over a p-adic field — after Sally-Tadić*

Abstract: In this talk, we will explain the classification for non-supercuspidal representations of GSp(4) over non-archimedean local field.

References :

1. Paul J. Sally, Jr. and Marko Tadić. *Induced representations and classifications for* $\mathrm{GSp}(2, F)$ *and* $\mathrm{Sp}(2, F)$, Mém. Soc. Math. France (N.S.) **52** (1993), 75–133.

2. Freydoon Shahidi, *A proof of Langlands' conjecture on Plancherel measures; complementary series for p-adic groups*, Ann. of Math. (2) **132** (1990), no. 2, 273–330.

3. I. N. Bernstein and A. V. Zelevinsky, *Induced representations of reductive $\mathfrak{p}$-adic groups. I*, Ann. Sci. École Norm. Sup. (4) **10** (1977), no. 4, 441–472.

4. J.-L. Waldspurger, *La formule de Plancherel pour les groupes p-adiques d'après Harish-Chandra*, Journal de l'Institut de Mathématiques de Jussieu **2** (2003), no. 2, 235–333.

Takuya Konno (Kyushu University):

Spectral decomposition of the automorphic spectrum of GSp(4)

Abstract: Let $\mathbb{A}$ be the adele ring of a number field F and write $G :=$ GSp(4). We explain the spectral decomposition of the right regular representation of $G(\mathbb{A})$ on $L^2(G(F)\mathbb{A}^\times \backslash G(\mathbb{A}))_\omega$ for a fixed central character ω.

References :

1. R. P. Langlands, On the functional equations satisfied by Eisenstein series, Springer LNM. 544, Springer-Verlag.

2. C. Moeglin and J-L. Waldspurger, Decomposition spectrale et series d'Eisenstein, Paraphrase sur l'ecriture, Progress in Mathematics, Vol.133, Birkhäuser.

3. T. Oda and J. Schwermer, *Mixed Hodge structures and automorphic forms for Siegel modular varieties of degree two*, Math. Ann. 286 (1990), pp. 481–509.

4. F. Shahidi, *A proof of Langlands conjecture on Plancherel measures; Complementary series for p-adic groups*, Ann. of Math. 132 (1990), pp. 273–330.

Tomonori Moriyama (Sophia University):

Representation theory of $\mathrm{GSp}(4, \mathbb{R})$ *with emphasis on discrete series*

Abstract: We plan to explain the following topics:

(1) Langlands classification and discrete series

(2) classification of discrete series (Blattner formula, global character, infinitesimal character, Langlands parameter)

(3) automorphic forms of discrete series type

(4) realization of discrete series

References :

Kyo Nishiyama's article (in Japanese), in *RIMS Kokyuroku* 909, titled *"Introduction to standard representation of semisimple Lie groups with emphasis on* $\mathrm{Sp}(2,\mathbb{R})$ *and* $\mathrm{SU}(2,2)$*"* which is available at

http://rtweb.math.kyoto-u.ac.jp/home_kyo/preprint/standard.pdf

will be very useful. Its reference section with his comments is also useful.

Besides there are standard text books such as:

1. A. W. Knapp, *Representation Theory of Semisimple Groups: An Overview Based on Examples*, Princeton University Press.

2. N. Wallach, *Real Reductive Groups* I, Academic Press.

2 Research Talks

Jim Brown (Ohio State University):
Saito-Kurokawa lifts and applications to arithmetic

Abstract: Let f be an elliptic newform of weight $2k-2$ and level N. The Saito-Kurokawa correspondence associates to f a cuspidal Siegel eigenform F_f. Building on work of Ribet and Wiles we show how one can use the Saito-Kurokawa correspondence to produce evidence for the Bloch-Kato conjecture for modular forms. In particular, I will present results of the form that if a prime divides $L(k,f)$ and satisfies various other hypotheses, then the prime also divides the order of the appropriate Selmer group.

Wee Teck Gan (University of California San Diego):
Restrictions of Saito-Kurokawa representations

Abstract: The (local) Gross-Prasad conjecture describes the restriction of representations from tempered L-packets of $\mathrm{SO}(n)$ to $\mathrm{SO}(n-1)$. It is natural to undertsand the extension of their conjecture to the situation of (non-tempered) Arthur packets. We examine this problem for the Saito-Kurokawa representations which are among the best-understood Arthur packets.

Tomoyoshi Ibukiyama (Osaka University):

Paramodular forms and compact twist: their dimensions and traces

Abstract: Any $\mathbb{Q}$-form of the compact twist of $\mathrm{GSp}(4)$ has three parahoric subgroups (at a ramified prime p), where $\mathbb{Q}$ is the rational number field. I proposed very precise conjectures on global correspondence with split $\mathrm{GSp}(4)$ for automorphic forms belonging to (each of) two of the parahoric subgroups in 1982 with good numerical evidence and dimensional equality in scalar value case. One of them concerns the paramodular group of level p in the split $\mathrm{GSp}(4)$. In this talk, I treat the vector valued Siegel modular forms belonging to the above paramodular group. We show the dimensional equality of the conjectured correspondence and partial results on coincidence (and exact values) of the trace of Hecke operators. This invokes two related problems to be solved: one is an injectivity of Ihara-lifting (the compact version of Saito-Kurokawa lifting obtained in 1963!) which is viewed as a space of old forms in the compact twist, and the other is a structure of some old forms of paramodular forms. Apart from this, since the vector valued case is more generic than the scalar valued case, we can see the matching of the contribution of conjugacy classes more definitely. *This is a joint work with Satoshi Wakatsuki.*

Taku Ishii (Chiba Institute of Technology):

Archimedean zeta integrals on $\mathrm{GSp}(4)$

Abstract: In this talk, we explain the papers of Moriyama (Amer. J. 2004) and Moriyama-Ishii (to appear) on the computation of the archimedean parts of Novodvorsky's zeta integrals for the spinor L-functions on $\mathrm{GSp}(4)$. By using the explicit formulas of Whittaker functions on $\mathrm{Sp}(4,\mathbb{R})$ studied by Oda and his collaborators, these integrals can be explicitly evaluated and the global functional equations and the entireness of the spinor L-functions are established. We also remark similar study for the standard L-function.

Hidenori Katsurada (Muroran Institute of Technology) and
Hisa-aki Kawamura (Hokkaido University):
Ikeda's conjecture on the Petersson product of the Ikeda lift

Abstract: For a cuspidal Hecke eigenform f of weight k with respect to $\mathrm{SL}(2, \mathbb{Z})$, let F be the Ikeda lift of f of degree n, and h the modular form in the Kohnen's plus space corresponding to f via the Shimura correspondence. Then T. Ikeda conjectured that the ratio of the Petersson products of F and h is expressed in terms of a product of certain L-values of f. In this talk, we explain our strategy for proving this conjecture, and report recent progresses, in particular, in case $n = 4$. As a related topic, we discuss congruences between Ikeda lifts and non-Ikeda lifts.

Kimball Martin (Columbia University):
Transfer from $\mathrm{GL}\,(2, D)$ *to* $\mathrm{GSp}\,(4)$

Abstract: It is known that there is a functorial transfer of generic automorphic representations of GSp(4) to GL(4), whose image is characterized by the nonvanishing of a certain period integral. There should be a similar transfer between GSp(4) and $\mathrm{GL}\,(2, D)$, where D is a quaternion algebra. We discuss joint work with H. Jacquet in which we use the relative trace formula to study period integrals on $\mathrm{GL}\,(2, D)$ and GL(4). Under certain hypotheses we prove that, for a cuspidal representation π of $\mathrm{GL}\,(2, D)$, the nonvanishing of a certain period integral implies that π transfers to GSp(4).

Hiro-aki Narita (Osaka City University):
Spinor L-functions of Arakawa lifting

Abstract: In this talk we discuss a global theta lifting from $\mathrm{O}^*(4)$ to $\mathrm{Sp}(1, 1)$, which was originally studied by Tsuneo Arakawa in the non-adelic setting. At the Archimedean place the lifting can be read as a theta correspondence between a discrete series of $\mathrm{O}^*(4)$ (isomorphic to $\mathrm{SL}_2(\mathbb{R}) \times \mathrm{SU}(2)$) and a quaternionic discrete series of $\mathrm{Sp}(1, 1)$. The aim of this talk is to give all non-Archimedean local factors (including local factors at ramified places) of the spinor L-functions for the lifting. A point of our result is that, in some case, all the local factors of the L-function at ramified places coincide with local factors of classical Hecke's L-functions of elliptic cusp forms. *This is a joint work with Atsushi Murase.*

Takeo Okazaki (Osaka University):
Proof of van Geemen-Nygaard-van Streaten's conjecture

Abstract: As a next step after Eichler-Shimura theory on elliptic modular forms, van Geemen, Nygaard, and van Starten studied the relation between some explicitly given 3-fold variety and Siegel modular forms. They gave some conjectures on the coincidence of their L-functions. We will explain the proof of their conjectures.

Brooks Roberts (University of Idaho):
Paramodular New- and Oldforms for GSp (4)

Abstract: In this talk we will present a complete theory of new- and oldforms for representations of GSp (4) over a nonarchimedean local field. These new- and oldforms are defined with respect to the paramodular groups, and representations admitting paramodular vectors include all generic representations, and many families of non-generic representations. We will describe the main theorems of the theory, and in particular, indicate how a newform determines, via its level, Atkin-Lehner eigenvalue, and Hecke eigenvalues, the degree four epsilon- and L-factors of the L-parameter of the representation. We will also try to describe some of the ideas and methods used in the proofs. One application of this theory is that it acts as a bridging mechanism between automorphic representations and modular forms. Another application is that, combined with results announced by Arthur and others, it implies a theory of Siegel paramodular newforms. A monograph containing the proofs of the results will be available at the conference.

Ralf Schmidt (University of Oklahoma):
Siegel Vectors for GSp(4)

Abstract: In the study of Siegel modular forms of higher level a prominent role is played by Siegel modular forms defined with respect to the Siegel congruence subgroups. These congruence subgroups are defined with respect to the usual congruence condition on the lower left block. We refer to such modular forms, and their local analogues, as Siegel vectors. Currently, there exists no good local or global theory of new- and oldforms for Siegel vectors. We will describe an initial decomposition of the space of Siegel vectors, and delineate a conjectural description of one component. We will also discuss work toward understanding the remaining components, and give examples.

Claus Sorensen (Princeton University):
Level-raising for GSp (4)

Abstract: I will discuss a result on level-raising for Saito-Kurokawa forms, and its application to the Bloch-Kato conjecture for classical modular forms. The idea is to first descend to an inner form G, which is compact at infinity, by theta correspondence. For G we then prove and apply a general result about raising the level. To transfer back to GSp (4) we use the stable trace formula. Consequently, we produce congruences between Saito-Kurokawa forms and certain stable automorphic forms.

David Whitehouse (Institute for Advanced Study):
On the transfer of automorphic representations from GSp (4) *to* GL (4)

Abstract: A recent paper of Arthur's describes a parameterization of representations of GSp (4) in terms of those of GL (4). These results should follow from a comparison of a stabilized trace formula for GSp (4) with a stabilized twisted trace formula for $\mathrm{GL}(4) \times \mathrm{GL}(1)$. We will describe the process of stabilization and the progress made towards the stabilization of these trace formulas.