ON THE LEVELS OF SPACES

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The level of a differential graded module over a differential graded algebra measures the number of steps to build the module in an appropriate triangulated category [1]. Using the notion, we have introduced a new topological invariant, which is called *the level of a space*, in [4]. In this talk, after describing fundamental properties of the invariant, computational examples of the levels of Borel constructions, including the Davis-Januszkiewicz spaces, are given. Here I would like to stress that such computations are made relying on (relatively) $\mathbb{K}$-formality of maps between $\mathbb{K}$-formal spaces; see [3], [6] and [7] for the formality.

Regrettably, what the level of a space measures is obscure. However it seems that the numerical invariant is related to L.-S. category of a DG module in a model category and to the inductive cocategory of a space in the sense of Ganea [2]. Considerations for the direction are in a further project.

The plan of this talk is as follows.

1. An overview of the triangulated category of DG modules over a DGA.
2. The levels of DG modules and of spaces
3. Fundamental properties of the levels of a spaces
4. Computational examples
5. Vistas (if time permits)

References

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