Study of Smith equivalent representations

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Abstract

Let G be a finite group and let $\mathfrak{S}$ denote the family of all smooth G-manifolds with exactly two G-fixed points whose underlying spaces are standard spheres. Two real G-modules V and W (of finite dimension) are called $\mathfrak{S}$-*related*, and written $V \sim_{\mathfrak{S}} W$ if there exists X in $\mathfrak{S}$ such that $T_a(X) \cong V$ and $T_b(X) \cong W$ as real G-modules for $a, b \in X^G$. Let

$$\mathrm{RO}(G, \mathfrak{S}) = \{[V] - [W] \in \mathrm{RO}(G) \mid V \sim_{\mathfrak{S}} W\}.$$

For $\mathcal{A} \subset \mathrm{RO}(G)$ and $\mathcal{F}, \mathcal{G} \subset \mathcal{S}(G)$, let

$$\mathcal{A}^{\mathcal{F}} = \{[V] - [W] \in \mathcal{A} \mid V^H = 0 = W^H \quad (\forall H \in \mathcal{F})\},$$

$$\mathcal{A}_{\mathcal{G}} = \{[V] - [W] \in \mathcal{A} \mid \mathrm{res}^G_H V \cong_H \mathrm{res}^G_H W \quad (\forall H \in \mathcal{G})\},$$

$$\mathcal{A}^{\mathcal{F}}_{\mathcal{G}} = (\mathcal{A}^{\mathcal{F}})_{\mathcal{G}}.$$

Let $\mathcal{P}(G)$ (resp. $\mathcal{P}(G)_{\mathrm{odd}}$) denote the set of subgroups P of G such that the order of P is a prime power (resp. an odd-prime power). For a prime p, let $\mathcal{N}_p(G)$ denote the set of normal subgroups N of G such that $|G/N| = 1$ or p.

Theorem 1. $\mathrm{RO}(G, \mathfrak{S})$ *is contained in* $\mathrm{RO}(G)^{\mathcal{N}_2(G)}_{\mathcal{P}(G)_{\mathrm{odd}}}$. *If a Sylow 2-subgroup of* G *is normal in* G *then*

$$\mathrm{RO}(G, \mathfrak{S}) \subset \bigcap_{p:\ prime \geq 5} \ \bigcap_{N \in N_p(G)} \left(\mathrm{RO}(G)^{\mathcal{N}_2(G) \cup \mathcal{N}_3(G) \cup \{N\}}_{\mathcal{P}(G)_{\mathrm{odd}}} \cup \mathrm{RO}(G)^{\mathcal{N}_2(G) \cup \mathcal{N}_3(G)}_{\mathcal{P}(G)_{\mathrm{odd}} \cup \{N\}} \right).$$

Theorem 2. $\mathrm{RO}(G, \mathfrak{S}) \smallsetminus \mathrm{RO}(G, \mathfrak{S})_{\mathcal{P}(G)}$ *is a finite set.*

Let G^{nil} denote the smallest normal subgroup N of G such that G/N is nilpotent.

Theorem 3. *Let* G *be an Oliver group such that* $G/G^{\mathrm{nil}} \cong C_3$. *Then* $\mathrm{RO}(G, \mathfrak{S})_{\mathcal{P}(G)}$ *is equal to* $\mathrm{RO}(G)^{\mathcal{N}_3(G)}_{\mathcal{P}(G)}$ *if a Sylow* 2*-subgroup of* G *is normal in* G*, and equal to* $\mathrm{RO}(G)^{\{G\}}_{\mathcal{P}(G)}$ *otherwise.*

We also discuss further results.