ON THE KERNEL OF THE MAGNUS REPRESENTATION OF THE AUTOMORPHISM GROUP OF A FREE GROUP

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Abstract. In this talk, we show that the abelianization of the kernel of the Magnus representation of the automorphism group of a free group contains a free abelian group of infinite rank.

Let F_n be a free group of rank $n \geq 2$, and $\mathrm{Aut}\, F_n$ the automorphism group of F_n. We denote by $\rho : \mathrm{Aut}\, F_n \to \mathrm{Aut}\, H$ the natural homomorphism induced from the abelianization $F_n \to H$. The kernel of ρ is called the IA-automorphism group of F_n, denoted by IA_n. The subgroup IA_n reflects many richness and complexity of the structure of $\mathrm{Aut}\, F_n$, and plays important roles on various studies of $\mathrm{Aut}\, F_n$.

In general, one of the most standard ways to study a group is to consider its representations. If a representation of the group is given, it seems to be important to determine how far it is from faithful. In this talk, we consider this matter for the Magnus representation

$$r_M : \mathrm{IA}_n \to \mathrm{GL}(n, \mathbf{Z}[H])$$

of IA_n.

Classically, the Magnus representation was used to study certain subgroups of IA_n. One of the most famous subgroup is the pure braid group P_n. The restriction of r_M to P_n is called the Gassner representation. It is known due to Magnus and Pelso that $r_M|_{P_n}$ is faithful for $n = 3$. Although the faithfulness of $r_M|_{P_n}$ has been studied for a long time by many authors, it seems, however, still open problem to determine it for $n \geq 4$. Another important subgroup is the Torelli subgroup $\mathcal{I}_{g,1}$ of the mapping class group of a surface. By a classical work of Dehn and Nielsen, the Torelli group $\mathcal{I}_{g,1}$ is considered as a subgroup of IA_{2g}. Suzuki showed that the restriction $r_M|_{\mathcal{I}_{g,1}}$ to $\mathcal{I}_{g,1}$ is not faithful for $g \geq 2$. Furthermore, by a recent remarkable work of Church and Farb, it is known that the abelianization of the kernel of $r_M|_{\mathcal{I}_{g,1}}$ is not finitely generated for $g \geq 2$.

From the facts as mentioned above, it is immediately seen that r_M itself is not faithful. There are, however, few results for the study of the kernel of r_M. Let $\mathcal{K}_n$ be the kernel of r_M. From our previous work, we showed that the abelianization $\mathcal{K}_n^{\mathrm{ab}}$ of $\mathcal{K}_n$ contains a certain free abelian group of finite rank. In this talk, we show that r_M is far from faithful. That is,

Theorem 1. *For $n \geq 2$, $\mathcal{K}_n^{\mathrm{ab}}$ contains a free abelian group of infinite rank.*

In this talk, we treat the case of $n \geq 3$. To prove the theorem, we consider some embeddings of $\mathcal{K}_n$ into the IA-automorphism group of a free group of higher rank. Then, taking advantage of the first Johnson homomorphisms, we detect infinitely many linearly independent elements in $\mathcal{K}_n^{\mathrm{ab}}$.