ON ISOVARIANT HOPF TYPE THEOREMS

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Let X and Y be G-spaces. A G-equivariant map $\varphi : X \to Y$ is called a G-isovariant map, if it preserves the isotropy groups, that is, $G_x = G_{\varphi(x)}$ holds for all $x \in X$. If a G-homotopy $F : X \times [0,1] \to Y$ between two isovariant maps $\varphi, \psi : X \to Y$ is G-isovariant, F is called a G-isovariant homotopy, and it is said that φ and ψ are G-isovariantly homotopic. We denote by $[X,Y]_G^{\mathrm{isov}}$ the set of G-isovariant homotopy classes of G-isovariant maps from X to Y.

The purpose of our talk is to give the structure of $[M, SW]_G^{\mathrm{isov}}$ for a free G-manifold M and a unitary G-representation sphere SW under the Borsuk-Ulam type inequality:

$$\dim M + 1 \leqq \dim SW - \dim SW^{>1},$$

where $SW^{>1} = \displaystyle\bigcup_{\{1\} \neq H \leq G} SW^H$.

Set $\mathcal{A} = \{H \in \mathrm{Iso}(W) | \dim SW^H = \dim SW^{>1}\}$ and $\mathcal{A}/G = \{(H) | H \in \mathcal{A}\}$. Let $w : G \to \{\pm 1\}$ be the orientation homomorphism defined by $w(g) = +1 \mathrm{or} -1$ whether $g \in G$ acts orientation-preservingly on M or not respectively. Let $\mathbb{Z}_w$ be the $\mathbb{Z}G$-module whose underlying module is $\mathbb{Z}$ and on which G acts by $g \cdot k = w(g)k$ where $k \in \mathbb{Z}_w$ and $g \in G$. Put $K_w = \ker w$ and

$$\mathcal{A}^+ = \{H \in \mathcal{A} | NH \leqq K_w\}, \quad \mathcal{A}^- = \{H \in \mathcal{A} | NH \nleqq K_w\}.$$

In the case of $\dim M + 1 < \dim SW - \dim SW^{>1}$, we see that all isovariant maps $f : M \to SW$ are isovariantly homotopic, that is, $[M, SW]_G^{\mathrm{isov}} = \{*\}$ holds. If $\dim M + 1 = \dim SW - \dim SW^{>1}$ and M is orientable, we have

$$[M, SW]_G^{\mathrm{isov}} \cong (\bigoplus_{(H) \in \mathcal{A}^+/G} \mathbb{Z}) \oplus (\bigoplus_{(H) \in \mathcal{A}^-/G} \mathbb{Z}_2).$$

While this result is shown by the equivariant obstruction theory, the correspondence to $(\bigoplus_{(H) \in \mathcal{A}^+/G} \mathbb{Z})$-term is interpreted as a degree-like function, which is called the multidegree of an isovariant map. Thus, our result is regarded as one of the isovariant versions of the classification theorem of Hopf.

In today's talk,we will also give some classification examples of isovariant maps by using our isovariant Hopf type theorem.

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