Abstract

"Integrable systems associated with character varieties"
M. Mulase (U. Calif., Davis)

Since the appearance of the seminal work of Hitchin, character varieties of Lie groups have been extensively studied from the point of view of geometry and representation theory. In this talk I will report a new result obtained in a joint work with Andrew Hodge that relates the Hitchin integrable systems for GL, Sp, and SO groups with the Sato Grassmannian and KP equations.

Recently the Hitchin integrable systems have received much attention in the study of geometric Langlands correspondence and Mirror Symmetry. Our recent work is motivated by our trial to understand the Langulands/Mirror duality in the Grassmannian context. Some of the recent developments in this fashionable area and geometry of character varieties will be briefly surveyed in the talk.

If time permits, another direction of research toward geometric structure of character varieties through integration theory on operator algebras may be presented. (This part is a joint work with Jerry Kaminker.

"Integral structures of toric quantum cohomology"
H. Iritani (Kyushu U.)

We study integral strucures in the space of solutions to quantum differential equaions, which yield tt*-geometry on quantum cohomology. We use mirror symmetry to find the "most natural" integral structure in quantum cohomology for toric orbifolds. If time permits, we also discuss a relationship between the integral structures and wall-crossings of quantum cohomology.

"Noncommutative Integrable Systems and Twistor Geometry"
M. Hamanaka (Nagoya U.)

I would discuss extension of integrable systems and soliton theories to non-commutative (NC) spaces. In string theory, there has been much development in NC gauge theories related to D-branes. NC gauge theories are equivalent to commutative gauge theories in background of magnetic fields. In particular, NC solitons correspond to (lower-dimensional) D-branes themselves and applied successfully to various analysis of D-branes dynamics. In this context, NC instantons and monopoles have been studied very intensively. The existence of the corresponding physical situations would also guarantee that of interesting mathematical structure of them.

On the other hand, there are many other integrable equations such as KP, KdV etc. Most of them seem to belong not to gauge theories but to scalar theories and perhaps has no physical correspondence. However we know that such integrable eqs. can be derived from only one equation, the anti-self-dual Yang-Mills (ASDYM) equation, in 4-dimensional spaces by reduction, which was first conjectured by Richard Ward and known as Ward's conjecture. Twistor theory is a good framework for discussion on integrability of ASDYM eq. and hence, via Ward's conjecture, gives a unified geometrical treatment of various integrable equations in diverse dimensions. (The results are summarized e.g. in the book of Mason and Woodhouse beautifully.)

In this talk, I would discuss NC ASDYM equations from the viewpoint of NC integrable systems and NC twistor theory. In particular, I would present a Backlund transformation for NC ASDYM eq. which yields various exact (Atiyah-Ward ansatz) solutions including NC instantons and NC non-linear plane waves. We have found that a kind of NC determinants, the quasideterminants (for a good survey, see [Gelfand and Retakh et. al, arXiv:math/0208146]), play crucial roles in construction of solutions. This is based on collaboration with Claire Gilson and Jonathan Nimmo (Glasgow).

I would also talk about NC version of Ward's conjecture [hep-th/0601209] (proposed first explicitly by me and Kouichi Toda five years ago [hep-th/0211148]) and derive various NC integrable equations from NC ASDYM eqs. by reduction. These equations actually have integrable-like properties such as infinite conserved equantities [MH, hep-th/0311206], exact N-soliton solutions e.g. [MH, hep-th/0610006] and so on. These results would lead to a kind of classfication of NC integrable equations from a geometrical viewpoint and applications to the corresponding physical situations and geometry also.

"Geometry of Painlevé equations (joint work with T. Uehara) "
K.Iwasaki (Kyushu U.)

Painlevé equations are usually studied from the view point of integrable systems. However we are able to show that some Painlevé equations are chaotic in a definitive sense, based on algebraic geometry, moduli theory, Riemann-Hilbert correspondence, geometry and dynamics on a character variety, ergodic theory of birational surface dynamics, and singularity theory.

"Quantum Painlevéquations"
H. Nagoya (Keio U.)

Painlevé equations are written into Hamiltonian systems with polynomial Hamiltonians in terms of canonical coordinates and have the affine Weyl group symmetries as Bäcklund transformation. We costruct a quantization of Painlevé equations with the affine Weyl group symmetries. We also explain the relation between our quantum Painlevé equations and that isomonodromic deformations and spaces of initial conditions.

"Real forms and finite order automorphisms of affine Kac-Moody algebras"
E. Heintze (U. Augsburg)

We give a new approach classifying real forms and finite order automorphisms of an affine Kac-Moody algebras and loop algebras. This approach works in the smooth as well as in the algebraic category and relates real forms and involutions (of the second kind) to pairs of finite dimensional symmetric spaces.

"Drawing the complex projective structures on once-punctured tori"
Y. Komori (Osaka City U.)

Asociated with the complex projective structure on Riemann surface X, we can consider the developing map and the holonomy representation of the fundamental group of X. Let $\mathsf{K}(\mathsf{X})$ be the set of complex projective structures on X whose holonomy groups are discrete. When X is a once-punctured torus, we can produce pictures of $\mathsf{K}(\mathsf{X})$ in the complex plane, which reflect the rich geometric structure of $\mathsf{K}(\mathsf{X})$. This is a joint work with Toshiyuki Sugawa (Hiroshima), Masaaki Wada (Nara) and Yasushi Yamashita (Nara).

"Noncompact Einstein homogeneous manifolds and geometric invariant theory"

J. Lauret (U. Cordoba)

The construction of Einstein metrics on manifolds is a classical problem in differential geometry and general relativity. A Riemannian manifold is called Einstein if its Ricci tensor is a scalar multiple of the metric. General existence and non-existence results are hard to obtain, and it is a natural simplification to impose additional symmetry assumptions, i.e. to consider metrics admitting a large Lie group of isometries.

We study Einstein manifolds admitting a transitive solvable Lie group of isometries (solvmanifolds). It is conjectured that these exhaust the class of noncompact homogeneous Einstein manifolds. J. Heber has showed that under certain simple algebraic condition (such a solvmanifold is called standard), Einstein solvmanifolds have many remarkable structural and uniqueness properties. In this paper, we prove that any Einstein solvmanifold is standard, by applying a stratification procedure from geometric invariant theory for reductive groups actions on projective algebraic varieties due to F. Kirwan.

The strata are parametrized by a finite set of elements in the Cartan subalgebra, which are (up to conjugation) the 'most responsible' directions for the instability of each vector in the stratum. Moreover, each stratum can be described in terms of semistable vectors for a suitable smaller action.

"CMC trinoids of genus $\mathsf{g} = 0$: a closer look"

J. Dorfmeister (TU Munich)

The generalized Weierstrass representation produces globally all CMC immersions from a Riemann surface into R^3. Therefore, if one is interested in a very specific type of immersion, one "only" needs to use the right type of Weierstrass data to obtain the desired type of immersion.

Such right Weierstrass data are known for some classes of immersions, like CMC-cylinders, CMC-tori, and CMC-trinoids of genus g =0 with embedded ends.(The latter ones are what will be called in this talk just simply "CMC-trinoids").

Following the generalized Weierstrass representation in detail, one notices that actually in addition to the Weierstrass data, which correspond to the coefficient matrix of some holomorphic/meromorphic ODE, also the initial conditions are of importance ("dressing").

In our setting, the Weierstrass data of some CMC-trinoid correspond to the necksizes of the asymptotic (Delaunay) ends. But which actual trinoid (with the same asymptotic Delaunay ends) one obtains depends on some initial conditions = some dressing. It turns out that the information, telling what type of trinoid of given necksizes on obtains, is encoded in the monodromy matrices corresponding to the ends.

In this talk, we will describe the monodromy matrices of CMC-trinoids in quite some detail. In particular, we will give a complete description of all monodromy matrices corresponding to rotationally symmetric CMC-troinoids.

"Real form surfaces of a complex constant mean curvature surface"

SP. Kobayashi (Tokyo Denki U.)

Complex constant mean curvature surfaces are by natural complexifications of constant mean curvature surfaces in Euclidean three space. In this talk we consider a converse construction, the real form surfaces of a complex constant mean curvature surface, which are obtained by the real forms of the Maurer-Cartan form for a complex constant mean curvature surface. We also give a unified theory for all real form surfaces via the generalized Weierstrass type representation, the so-called DPW method.

"Surfaces in three-dimensional Lie groups"
I. Taimanov (Inst. Math., Novosibirsk)

We derive the Weierstrass (or spinor) representation for surfaces in three-dimensional Lie groups. In some particular cases (Nil, $\widetilde{\mathrm{SL}}_2$, and Sol geometries) we establish the generating equations for minimal surfaces. By using the spectral properties of the corresponding Dirac operators we find analogs of the Willmore functional for surfaces in these geometries and demonstrate their relation to isoperimetric problems.

"The moduli space of harmonic 2-spheres in round spheres"
J. Bolton (U. Durham)

It is with some pleasure that I return to this topic! I would like to report on recent work by Luis Fernandez, John C Wood and myself on the above moduli space M. Firstly, Fernandez has proved a conjecture of myself and Lyndon Woodward concerning the dimension of M. Secondly, in the case when the target is the unit 4-sphere, Wood has considered the integrability of Jacobi fields along elements of M, Fernandez and myself have considered the regularity of M, and we have also investigated elements of M possessing some symmetry.

"Knots and minimal surfaces"
T. Tanaka (Osaka City U.)

A knot is an embedded circle in Euclidean 3-space. We show that every knot is ambient isotopic to a knot which bounds a stable embedded minimal surface in 3-space. We also study embedded minimal surfaces with boundaries in 4-space.

"Douglas's approach to solving the Plateau problem"
M. Micallef (U. Warwick)

In this talk, I shall challenge the popular belief that Douglas arrived at his mysterious functional for solving the Plateau Problem by direct consideration of Dirichlet's integral and its relation to the area functional. I shall describe how, by looking at abstracts of Jesse Douglas in the Bulletin of the American Mathematical Society, I have been able to infer how Douglas MAY have arrived at his functional. Douglas was awarded one of the first Fields Medals for his work on the Plateau problem. I shall describe some of the amusing aspects of the Fields Medal ceremony at which Douglas was awarded his prize.

This is a joint work with the mathematical historian Jeremy Gray.

"Quantum cohomology of weighted projective spaces"
H. Sakai (Tokyo Metrop. U.)

We study the differential equation associated to the orbifold quantum cohomology of a weighted projective space, following work of Coates-Corti-Lee-Tseng. We describe how the D-module (Dubrovin connection) contains all of the important geometrical information about the orbifold quantum cohomology. (Joint work with Martin Guest.)

"Integrable fields on manifolds with SU(3)-structure"
Y. Machida (Numazu Nat. Coll. Tech.)

It is well known that an integrable SU(3)-structure defines a complex 3-dimensional Ricci-flat Kähler, i.e., Calabi-Yau manifold. In this lecture, we deal with non-integrable SU(3)-structures, in particular, 6-dimensional nearly Kähler structures. This structure has relation to type II geometry in superstring theory. We discuss integrable fields: abel gauge fields (U(1)-instantons), non-abelian gauge fields (SU(3)-instantons), gravitational fields (nealy Kähler metrics), associated with the SU(3)-structure.

"Soliton deformations of surfaces "

I. Taimanov (Inst. Math., Novosibirsk)

We discuss soliton deformations of surfaces via modified Novikov-Veselov and Davey-Stewartson equations, show that they act geometrically on tori with many conservation laws given by the spectral curves of linear operators associated to these soliton hierarchies.