The Cauchy Problem of the Ward Equation:

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Taking a dimension reduction and a gauge fixing of the self-dual Yang-Mills equation in the space-time with signature $(2,2)$, one derives a $2+1$ dimensional $SU(N)$ chiral field equation with an additional torsion term.

$$\begin{aligned}&-\left(J^{-1}J_t\right)_t+\left(J^{-1}J_x\right)_x+\left(J^{-1}J_y\right)_y+\nu_0\left\{\left(J^{-1}J_y\right)_x-\left(J^{-1}J_x\right)_y\right\}\\&+\nu_1\left\{\left(J^{-1}J_t\right)_y-\left(J^{-1}J_y\right)_t\right\}+\nu_2\left\{\left(J^{-1}J_x\right)_t-\left(J^{-1}J_t\right)_x\right\}=0.\end{aligned}$$

Where J lies in $SU(N)$ and $\nu=(\nu_0,\nu_1,\nu_2)$ is a constant unit vector. Letting $\nu=(1,0,0)$ (time-like) and $\nu=(0,1,0)$ (space-like), we obtain two integrable systems, the 3-dimensional relativistic-invariant system and the Ward equation.

One important class of solutions for integrable systems are solitons of which the associated eigenfunctions $\psi(x,y,t,\lambda)$ are λ-rational functions. The construction of simple solitons, and the study of their scattering properties was done by [4] for the 3-dimensional relativistic-invariant system and by many mathematicians for the Ward equation, see [1] for references.

Besides, mathematicians study the inverse scattering problem and solve the Cauchy problem of the 3-dimensional relativistic-invariant system [4], [5] and of the Ward equation [6], [3], [2] if the initial potential is sufficiently small. Under the small data condition, the associated eigenfunction $\psi(x,y,t,\lambda)$ is λ-holomorphic outside a contour in the complex plane. Therefore this class of solutions does not include solitons in previous study.

Our main contribution is solving the inverse scattering problem and the Cauchy problem of the Ward equation without small data constraints.

References

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