On an eigenvalue problem related to the critical Sobolev exponent: variable coefficient case

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Let $\Omega \subset \mathbb{R}^N (N \geq 3)$ be a smooth bounded domain, $p = (N+2)/(N-2)$, $c_0 = N(N-2)$, $\varepsilon > 0$ and $p_\varepsilon = p - \varepsilon$. $K \in C^2(\overline{\Omega})$, $K > 0$ is a given function.

In this talk, we are concerned with some spectral properties of least energy solutions u_ε to the problem

$$(P_{\varepsilon,K}) \begin{cases} -\Delta u = c_0 K(x) u^{p_\varepsilon} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

when $\varepsilon > 0$ is small.

Let us consider the linearized eigenvalue problem around the least energy solution u_ε to $(P_{\varepsilon,K})$:

$$(E_{\varepsilon,K}) \begin{cases} -\Delta v_{i,\varepsilon} = \lambda_{i,\varepsilon} \left(c_0 p_\varepsilon K(x) u_\varepsilon^{p_\varepsilon - 1} \right) v_{i,\varepsilon} & \text{in } \Omega, \\ v_{i,\varepsilon} = 0 & \text{on } \partial\Omega, \\ \|v_{i,\varepsilon}\|_{L^\infty(\Omega)} = 1 \end{cases}$$

for $i \in \mathbb{N}$. Under some assumptions of K (including the nondegeneracy of its maximum point as a critical point of K), we prove precise asymptotic estimates for the first $(N+2)$ eigenvalues and eigenfunctions of $(E_{\varepsilon,K})$ as $\varepsilon \to 0$. These results correspond to the ones recently obtained by Grossi and Pacella (Math.Z., 2005) for the "one-point blow up solutions" to $(P_{\varepsilon,K})$ with $K \equiv 1$.